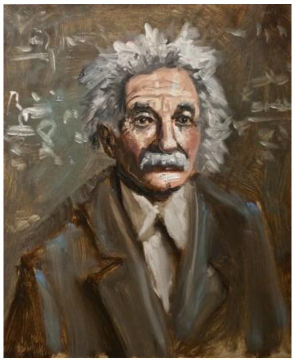
WHAT IS MATH?

Introduction

Every person has taken math in school some time in their life. BUT, why don't we have a clear definition of Mathematics? What on earth are we studying? What kind of education puts you through 12 years of math and doesn't even explain to you what it is?

Galileo Galilei: "If I were again beginning my studies, I would follow the advice of Plato and start with mathematics."

Albert Einstein: "Pure mathematics is, in its way, the poetry of logical ideas."



What you need to know to understand this lesson:

You don't need to know anything! You just need a Rational Mind.

Why you should bother know what math is:

1. Because it's the most precise and accurate language ever.
2. Because it's the most systematic and logical branch of knowledge humans ever conceived of.
3. Because Science and Engineering are ridiculously vague without it.
4. Because you have a course on it and you want to actually understand it?

Math as a branch of knowledge:

Before you learn any branch of knowledge, you have to understand that every knowledge is “knowledge OF something” not just knowledge period. This “something” is a form of Being (with a unique set of states) and has different forms of Becoming (changes of states). This branch of knowledge that attempts to understand Being and Becoming is called Ontology.

Thus: what we ought to understand is, what are the objects of mathematical knowledge? So what is mathematics the knowledge of?

And then we ought to know the Method we come to KNOW these different beings and becomings. How do we acquire knowledge about these entities? What fields and topics does this Method cover and what are the limits of this unique Method of Knowing?

Thus: what we ought to understand is, what are the methods of mathematical reasoning and knowledge? So what is the form of knowing we use when doing mathematics?

Approach

We will approach this by discussing five distinctions in ontology and epistemology:

1. Abstract vs Concrete entities
2. Idealism vs Realism
3. Rational vs Empirical evidence
4. Deductive vs Inductive reasoning
5. Synthetic vs Analytic statements

The Abstract VS Concrete Entities Distinction

Plato: "Geometry existed before creation."

A concrete entity refers to a thing that exists physically.

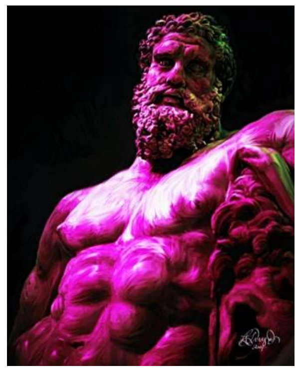
An abstract entity refers to a thing that is not necessarily physical. It is a “type of thing” or just an “idea”.

Physical means within space and time. So: abstract entities don’t necessarily have an existence in space and time.

Note that abstract ideas may or may not have their concrete particular counterparts: for example the idea of a snake is an abstract idea, but that doesn't mean that there are no snakes concretely, the idea of a unicorn is an abstract idea, but that doesn't mean there must be some unicorn somewhere concretely.

Zeus doesn't exist. But is he still all powerful? Well the abstract idea of Zeus entails by definition an all-powerful Being. BUT, there is no concrete Zeus who actually exists, thus Zeus is concretely non-existing, therefore you can't say he is or is not all-powerful concretely.

Zeus: "Lies, AM ALL POWERFUL, just look at my six pack."

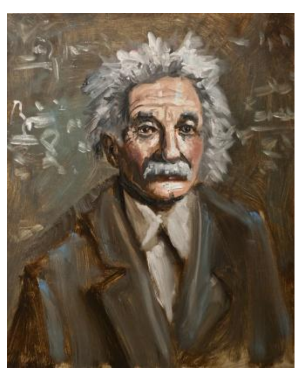


Mathematical entities are Abstract entities that may or may not have concrete counterparts. On the other hand, scientific entities are concrete entities that we can study abstractly. Thus in mathematics we start with abstract entities (ie: real and complex numbers), then we can if we want, look in the world and see where they exist concretely. On the other hand, in science we start with concrete entities (ie: solids and fluids), then we can abstract away from their concreteness into the roam of mathematics.

Note that when I say science, I mean the scientific method not science as vaguely "knowledge" only.

The Idealism VS Realism Distinction

Albert Einstein: "How can it be that mathematics, a product of human thought independent of experience, is so admirably adapted to the objects of reality?"



Idealism in metaphysics is the position that what you know about an entity is dependent on the mind. Thus what you know about an entity does not necessarily exist independent of your mind.

Realism in metaphysics is the position that what you know about an entity is dependent on the entity. Thus what you know about an entity exists independent of your mind.

It *seems* that our knowledge of mathematical entities and their properties is dependent on our minds rather than these entities by themselves. For abstract entities are entirely dependent on the mind conceiving them, not their actual existence in the real world. Thus we can go back to Euclid's definitions and axioms and alter them, and our new systems of geometry won't be more true or false than the old systems. Both systems are correct as abstract systems. Some of them may be more or less applicable to the concrete world but that doesn't mean they are wrong.

On the other hand, scientific entities and their properties *seem* to have an existence of themselves independent of our minds. For they are initially concrete entities in the physical world which seems to follow its own rules rather than our own rules. Thus if we look at scientific knowledge, if we alter Newton's theories of gravity due to new observations and experiments, our new understanding of gravity is indeed more TRUE and CORRECT than the old understanding.

Note that the word real is not used in opposition to the word fake or illusions or deceit. That is, ideal entities that we can't directly observe like space, time, gravity are not more or less "real" than a table that exists in space and time which is affected by gravity. It is just a different mode of being.

The Rational VS Empirical Distinction

Rational knowledge is knowledge independent of our senses. A claim that is "a priori" is independent of experience.

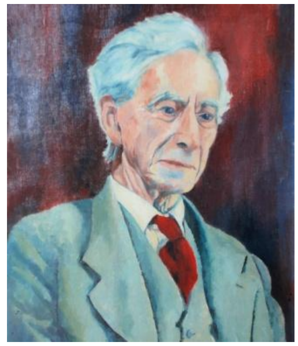
Empirical knowledge is knowledge dependent on our senses. A claim that is "a posteriori" is dependent on experience.

Mathematical knowledge seems to be entirely independent from our senses. We might get inspired through observing things in nature to study geometry for example, but all geometric claims/propositions/theorems are entirely independent of sense data. Thus mathematical propositions are all a priori.

On the other hand, scientific knowledge seems to be entirely dependent on our senses. For without any observation in nature or through experiments no scientific theory would be. Observation means that we are collecting sense data. Of course the analysis of sense data is entirely rational, but the point is: you cannot have a scientific claim/proposition/theorem that is entirely independent of sense data. Thus scientific propositions are all a posteriori.

The Deductive VS Inductive Distinction

Bertrand Russell : "Mathematics takes us still further from what is human, into the region of absolute necessity, to which not only the actual world, but every possible world, must conform."



Alfred North Whitehead: "Mathematics, in its widest significance, is the development of all types of formal, necessary, deductive reasoning."

Logic is the method of connecting premises to conclusions using a set of laws. It is a method of using some knowns (premises) to know some unknowns (conclusions). In order for a statement to be true, its premises must be true, and its logic must be true (following the laws of logic). If a statement's premises are false then we say the statement is incoherent. It's as if you took wrong data and analyzed it perfectly, even though the analysis was perfect the conclusion is still false. If a statement's logic is false then we say the statement is inconsistent. It's as if you took perfect data and analyzed it in a wrong way, even though the data was perfect the conclusion is false.

Logic is only considered with linking premises to conclusions. The truth of premises depends on epistemology/ontology only, unless the premises were derived through more logic and argument.

Deductive reasoning is a specific type of logic where the conclusions NECESSARLY follow from the premises. Necessary truths mean that there is no uncertainty or probability involved, if the premises "all men are mortal; Socrates is a man" are true, it is necessarily true that "Socrates is mortal".

Inductive reasoning is a specific type of logic where the conclusions CONTINGENTLY follow from the premises. Contingent truths mean that there is some uncertainty or probability involved. That is, some conditions must occur in order for the conclusion to follow. For example, given the premises "we heated 1000 metals, they all expanded" does not necessarily mean that "ALL metals expand when heated", it is indeed probable that all metals expand when you heat them, but its not necessary. Why? Because we only sampled 1000 metals from a practically infinite population of metals in the universe only on earth at this point in space and time.

Mathematical reasoning is deductive reasoning with necessary conclusions.

Scientific reasoning is inductive reasoning with contingent conclusions.

The Synthetic VS Analytic Distinction

Analytic propositions get their meaning from themselves. For example “all bachelors are single” is an analytic statement, by definition of bachelors we understand that these people who are bachelors are single.

Synthetic propositions get their meaning from the world. For example “all bachelors are unhappy” is a synthetic statement, it is not by definition of bachelors that we understand that bachelors are unhappy, rather it is through looking at the world that we get such an idea (note: this is just an example, it’s obviously a false statement and bachelors can be happy)

In mathematics, propositions are sometimes analytic but mostly synthetic. So when we say “12=3+9” we are getting the meaning that 12 equals 3+9 not from the definition of 12 but from how the world is. Thus no matter how you analyze the concept “3+9” you won’t find 12 setting there in that concept. BUT if you rely on your intuition or start representing 3 and 9 by your fingers, you see that it becomes 12. Or if we say “the shortest path between two points in space is a straight line”, no matter how much you analyze the concept “two points in space” you won’t find the concept “straight line is the shortest path” contained by it. But if you rely on your intuition or start representing points in space and measuring distances you will see that the shortest path is the straightest one.

In science, propositions are also sometimes analytic but mostly synthetic. Typically the synthetic ones are the useful one. For example “F=ma”, no matter how much you analyze the concept of F, you won’t find ma hidden in the concept, but when you look out in the world, you will find that indeed F equals ma. Even though the concept of forces does not by definition contain in it the concepts of the product of mass an acceleration.

Summary

Mathematics is the study of necessary synthetic truths through rational deductive reasoning on abstract ideal entities.

Science is the study of contingent synthetic truths through empirical inductive reasoning on concrete real entities.

Sub-divisions of Mathematics

Each branch of knowledge has subdivisions, for example, one way to subdivide mathematics is studying it as:

- Pure mathematics:
  - Quantity (like numbers and arithmetic)
  - Structure (like functions and algebra)
  - Space (like Euclidean or non-Euclidean geometry and topology)
  - Change (like calculus)
- Applied mathematics
  - Computational methods (like approximation and numerical analysis)
  - Statistics (like hypothesis testing and certainty analysis)

But this is not very useful, for calculus is all about change, but it needs understanding of numbers, functions, algebra, lines and areas, etc… Thus I will not discuss the different branches of mathematics here.

Why one ought to know this? (AGAIN!)

Because when you study physics, and you go through all the course materials on earth, ace every exam, but you don't understand the observations and experiments (empirical inductive reasoning) and the concrete entities you are studying, you are not a physicist nor do you have a physicist's understanding. In other words: you missed the point of studying physics.

Because when you study math, and you go through all the course materials on earth, ace every exam, but you don't understand the rational deductive reasoning that causes each theorem to be true, you are not a mathematician nor do you have a mathematician's understanding. In other words: you missed the point of studying mathematics.

The same can be said about each branch of knowledge including engineering. I didn't want to get more examples than mathematics and science to keep this lesson at a reasonable simplicity.

Schools and universities don't teach this way because their goal is not to produce Leibniz or Newton or Da Vinci, their goal is to produce tamed domesticated animals to feeds the production forces at the given time and place.

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